

0.1 - Factoring and Operations with Rational Expressions

Factor each expression in #1-10.

1. $x^4 - 9x^2$ 2. $x^2 + 4x - 32$ 3. $x^4 + 4x^2 - 32$ 4. $x^6 + 4x^3 - 32$ 5. $6x^2 - x - 2$

6. $10x^2 - 31x + 15$ 7. $x^3 - 2x^2 - 7x + 14$ 8. $x(x-4)^3(x-2) - 3(x-4)^2(x-2)^2$

9. $(x+1)^3(x^3+8x) - 8(x+1)^4$ 10. $6x(x-2)^2(2x^2+1)^2 + 2(2x^2+1)(2-x)^3$

Simplify each rational expression in #11-17.

11. $\frac{9-x^2}{x^3-27}$ 12. $\frac{3x^2-17x+20}{6x^2-7x-5}$ 13. $\frac{x^2+x+1}{x^6-1}$

14. $\frac{(x+6)^4(x-3)^4 - (x-3)^3(x+6)^5}{3(3-x)^2}$ 15. $\frac{x^2-6x+9}{x^2-1} \cdot \frac{2x^2-2x}{x-3}$

16. $\frac{4x^2-9}{2x^2-13x+15} \div \frac{4x^2-1}{2x^2-11x+5}$ 17. $\frac{7+6x-x^2}{3x-2} \cdot \frac{2x-3}{x^2-8x+7} \div \frac{1-x^2}{x^2-2x+1}$

Answers:

1. $x^2(x-3)(x+3)$ 2. $(x+8)(x-4)$ 3. $(x^2+8)(x-2)(x+2)$ 4. $(x^3-4)(x+2)(x^2-2x+4)$

5. $(3x-2)(2x+1)$ 6. $(5x-3)(2x-5)$ 7. $(x-2)(x^2-7)$ 8. $(x-4)^2(x-2)(x-6)(x-1)$

9. $(x+1)^3(x-2)(x^2+2x+4)$ 10. $4(x-2)^2(2x^2+1)(3x^3+x+1)$ 11. $\frac{-(3+x)}{x^2+3x+9}$ 12. $\frac{x-4}{2x+1}$

13. $\frac{1}{(x-1)(x+1)(x^2-x+1)}$ 14. $-3(x+6)^4(x-3)$ 15. $\frac{2x(x-3)}{x+1}$ 16. $\frac{2x+3}{2x+1}$ 17. $\frac{2x-3}{3x-2}$

0.2 - Simplifying Expressions and Solving Equations

Simplify each expression.

$$1. \frac{5}{x-1} + \frac{8}{(x-1)^2} - \frac{3}{(x-1)^3}$$

$$2. \frac{3x-2}{x-1} + \frac{2}{1-x} - \frac{x}{1-x}$$

$$3. \frac{x}{3+x} - \frac{x}{3-x} - \frac{x^2}{x^2-9}$$

$$4. 2x+1 - \frac{6x^3+x^2-1}{2x-1}$$

$$5. \frac{x^2+1}{x-1} - x+1 - \frac{x^2-1}{1-x} + \frac{x^3+x+4}{x^2-1}$$

$$6. \frac{6}{1-3x} - \frac{1}{2x-1} + \frac{3}{x} + \frac{x}{5x-6x^2-1}$$

$$7. \frac{x+3}{x^2+5x+6} + \frac{x+2}{x^2+8x+12} - \frac{3}{x+6}$$

$$8. \frac{1 + \frac{2}{x^2} + \frac{1}{x^4}}{1 + \frac{2}{x} + \frac{1}{x^2}}$$

$$9. \frac{x^2 - \frac{1}{x}}{x + \frac{1}{x} + 1}$$

$$10. \frac{\frac{2x+h}{x+h} + 1}{\frac{2x+h}{x+h} - 1}$$

$$11. \frac{4 - \frac{1}{1-x}}{16 + \frac{7}{x^2-1}}$$

Solve for real values of x - give answers in exact form.

$$12. x - \frac{2x-1}{3} = \frac{3x-5}{5}$$

$$13. 7x - 4(x-6) = 3(x-8)$$

$$14. 4x^2 - 9x + 2 = 0$$

$$15. 2x^2 + 3 = 6x$$

Answers:

$$1. \frac{5x^2-2x-6}{(x-1)^3} \quad 2. 4 \quad 3. \frac{x^2}{(x+3)(x-3)} \quad 4. -3x^2 \quad 5. \frac{(2x+3)(x^2+1)}{(x+1)(x-1)} \quad 6. \frac{2x^2-8x+3}{x(2x-1)(3x-1)}$$

$$7. \frac{-x+2}{(x+6)(x+2)} \quad 8. \frac{(x^2+1)^2}{x^2(x+1)^2} \quad 9. x-1 \quad 10. \frac{3x+2h}{x} \quad 11. \frac{x+1}{4x+3}$$

$$12. 5 \quad 13. \text{All real numbers} \quad 14. \frac{1}{4}, 2 \quad 15. \frac{3 \pm \sqrt{3}}{2}$$

0.3 - Factoring, Simplifying, Operations with Rational Expressions, Linear Equations

Factor each expression completely.

- $2x^7 - 128x$
- $10x^6 + 17x^3 + 6$
- $4x^4 - 37x^2 + 9$
- $x(x-3)^4 + 4(3-x)^3$
- $5x^3(x-5)^4(2x+1) - 10x^2(x-5)^3(2x+1)^2$

Simplify each expression.

- $\frac{x^3 - 6x^2 + 9x}{(3-x)^4}$
- $\frac{x^3 + 8}{(2-x)^3} \cdot \frac{x^2 - 4x + 4}{x^4 - 16}$
- $\frac{1 - \frac{6}{x} + \frac{5}{x^2}}{\frac{1}{x^2} - \frac{5}{x^3}}$
- $\frac{1 + \frac{2}{x-1}}{\frac{x^2 + x}{x^2 + x - 2}}$
- $\frac{\frac{1}{x-2} - \frac{1}{x-3}}{1 + \frac{1}{x^2 - 5x + 6}}$
- $\frac{x}{2-3x} + \frac{2x}{3x+2} - \frac{2-7x}{9x^2-4}$
- $\frac{1}{x+2} - \frac{2x+9}{6+x-x^2} - \frac{2x}{x^2-2x-3}$

Give the equation of each line described in #13-21.

- through (2, 8) and (7, 1)
- through (8, 2) and the origin
- through (5, 6) and vertical
- parallel to $y = 5x - 6$, through (8, 4)
- perpendicular to $y = 5x - 4$, through (2, 2)
- through (0, 7) and (4, -6)
- through (5, 6) and horizontal
- with x-intercept 3 and y-intercept 9
- parallel to $5x - y = 13$, through (6, -4)

Answers:

- $2x(x-2)(x+2)(x^2-2x+4)(x^2+2x+4)$
- $(5x^3+6)(2x^3+1)$
- $(x-3)(x+3)(2x+1)(2x-1)$
- $(x-3)^3(x-4)(x+1)$
- $5x^2(x-5)^3(2x+1)(x^2-9x-2)$
- $\frac{x}{(x-3)^2}$
- $\frac{-(x^2-2x+4)}{(x-2)^2(x^2+4)}$
- $x(x-1)$
- $\frac{x+2}{x}$
- $\frac{-1}{x^2-5x+7}$
- $\frac{x+1}{3x+2}$
- $\frac{x+3}{(x-3)(x+1)}$
- $y-1 = -\frac{7}{5}(x-7)$ or $y-8 = -\frac{7}{5}(x-2)$ or $7x+5y=54$
- $y-7 = -\frac{13}{4}x$ or $y+6 = -\frac{13}{4}(x-4)$ or $13x+4y=28$
- $y-2 = \frac{1}{4}(x-8)$ or $y = \frac{1}{4}x$ or $x-4y=0$
- $y=6$
- $x=5$
- $y-9 = -3x$ or $y = -3(x-3)$ or $3x+y=9$
- $y-4 = 5(x-8)$ or $5x-y=36$
- $y+4 = 5(x-6)$ or $5x-y=34$
- $y-2 = -\frac{1}{5}(x-2)$ or $x+5y=12$

0.3.1 - Logarithm and Exponential Expressions and Equations

Simplify each expression.

1. $\log_2 16$

2. $\ln e^5$

3. $\log_5 1$

4. $4^{\log_4 3}$

5. $\log_3 \sqrt{27}$

6. $10^{2\log 5}$

7. $\ln 1$

8. $\log_3 \left(\frac{1}{81} \right)$

9. $e^{\ln 4}$

10. $\log 10$

11. $\ln e$

12. $\log_4 4$

13. $\ln \sqrt{e}$

14. $\log_{49} \sqrt[3]{7}$

15. $\log_3 3^5$

16. $\log_4 2 + \log_4 8$

17. $\log_4 128 - \log_4 8$

Solve each equation for x.

18. $\log_2 x = 5$

19. $\log_9 x = \frac{3}{2}$

20. $\log x + \log(x-3) = 1$

21. $5^{3x-1} = 25^{x+1}$

22. $4^{2x-1} = 8^{\frac{x}{3}}$

23. $2^x = 5$

Answers:

1. 4

2. 5

3. 0

4. 3

5. $\frac{3}{2}$

6. 25

7. 0

8. -4

9. 4

10. 1

11. 1

12. 1

13. $\frac{1}{2}$

14. $\frac{1}{6}$

15. 5

16. 2

17. 2

18. 32

19. 27

20. 5

21. 3

22. $\frac{2}{3}$

23. $\frac{\ln 5}{\ln 2}$ or $\log_2 5$

0.3.2 - Trigonometric Expressions and Equations

Determine the exact value of each expression.

1. $\sin(60^\circ)$ 2. $\cos\left(\frac{\pi}{4}\right)$ 3. $\sec(30^\circ)$ 4. $\tan\left(\frac{\pi}{4}\right)$ 5. $\csc\left(\frac{\pi}{6}\right)$

6. $\cos\left(\frac{3\pi}{4}\right)$ 7. $\tan\left(\frac{7\pi}{6}\right)$ 8. $\sin(3\pi)$ 9. $\sec(270^\circ)$ 10. $\sin\left(-\frac{\pi}{6}\right)$

Find the exact value of x for $0 \leq x \leq 360^\circ$.

11. $\cos x = -\frac{\sqrt{3}}{2}$ 12. $\tan x = 1$ 13. $\sin x = \frac{\sqrt{3}}{2}$ 14. $\csc x = -\sqrt{2}$

Find the exact value of x for $0 \leq x \leq 2\pi$.

15. $\cot x = -\frac{1}{\sqrt{3}}$ 16. $\sin x = -\frac{1}{2}$ 17. $\cos x = 0$ 18. $\sec x = -\frac{2}{\sqrt{3}}$

Use a calculator to find the value, to three decimal places, of x for $0 \leq x \leq 360^\circ$.

19. $\sin x = -\frac{1}{4}$ 20. $\tan x = -0.65$ 21. $\sec x = 3$ 22. $\cot x = \frac{3}{2}$

Answers:

1. $\frac{\sqrt{3}}{2}$ 2. $\frac{1}{\sqrt{2}}$ 3. $\frac{2}{\sqrt{3}}$ 4. 1 5. 2 6. $-\frac{1}{\sqrt{2}}$ 7. $\frac{1}{\sqrt{3}}$

8. 0 9. Undefined 10. $-\frac{1}{2}$ 11. $x = 210^\circ, 330^\circ$ 12. $x = 45^\circ, 225^\circ$

13. $x = 60^\circ, 120^\circ$ 14. $x = 225^\circ, 315^\circ$ 15. $x = \frac{2\pi}{3}, \frac{5\pi}{3}$ 16. $x = \frac{7\pi}{6}, \frac{11\pi}{6}$

17. $x = \frac{\pi}{2}, \frac{3\pi}{2}$ 18. $x = \frac{5\pi}{6}, \frac{7\pi}{6}$ 19. $x = 194.478^\circ, 345.522^\circ$

20. $x = 146.976^\circ, 326.976^\circ$ 21. $x = 70.529^\circ, 289.471^\circ$ 22. $x = 33.690^\circ, 213.690^\circ$